Parametric identification of damaged dynamic systems with hysteresis and slip

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Parametric identification of damaged dynamic systems with hysteresis and slip

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Abstract. The improvement of performance-based seismic design techniques requires the understanding of the overall behaviour of a structure up to collapse. Therefore, both experimental data and analytical models are needed. In a greater detail, material and geometrical nonlinearities, strength degradation and stiffness deterioration should be identified. Also in view of probabilistic-based simulations, hysteretic models, i.e. models able to describe with accuracy the hysteretic and degrading nature of the dynamic response of structures, have to be striven for. At higher level of excitation or under seismic loading, civil structures can exhibit slip effects. This paper presents a technique for the identification of non-linear in framed systems. A new a model for simulating slip effects is also introduced, which demonstrated to be suitable for identification purposes.

1. Introduction
The importance of non-linear identification is increasing rapidly in the civil engineering field. In earthquake engineering in fact, performance based design calls for expensive experimental tests, whose outcomes, in terms of strength, ductility and dissipation properties, should be assimilated by non-linear and/or time-varying models.

Methods for identifying hysteretic systems can be subdivided into parametric and non parametric approaches. In the former case, an a priori selection of a specific model is needed and the identification reduces to determining the coefficients of this model. In the latter, non-parametric methods, like neural networks, do not require any assumption on type and localisation of structural non-linearities, but identified quantities cannot be directly correlated to the equations of motion. Classical non-parametric methods are generally based on the extension of the restoring force surface method. Along that line, Benedettini et al., [1] approximated the surface of the time derivative of a restoring force with a set of basis functions including the velocity and the force itself. Masri et al. [2] extended this approach by including displacements in the set of basis functions. Moreover, we deem worth to mention: i) the recent on-line approaches by Wu & Smyth [3], who successfully applied the Unscented Kalman Filter technique; ii) the technique suggested by Ceravolo et al. [4], which is based

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on a Volterra series representation of a non-linear time-varying system. The latter strategy has been originally developed by the authors melting the concept of non-linear identification introduced by the *Restoring Force Surface* (RFS) method [5,6] to the concept of instantaneous identification [7]. A non-parametric version of this approach has been recently applied to identify the damage in a steel-concrete frame subject to pseudo-dynamic tests [8].

In this study, in view of a purely parametric approach, a new model is introduced to simulate the response of hysteretic systems in presence of slip effects, which is loosely inspired to previous studies conducted by Baber and Noori [9]. A few numerical examples concerning the non-linear identification of a framed structure under sweep-sine and earthquake excitation will be finally presented.

2. Models for systems to slip and hysteresis

In this section the authors introduce a model which is representative of a hysteretic behaviour in presence of slip effect. The Bouc-Wen-Baber-Noori model [9] is retrieved and modified by introducing a sigmoid function as source of slip, and generalising the model to a 2 Degrees of Freedom (2-DoF) system. To start with, the slip non-linear effect is assumed to be in series with the linear stiffness (Figure 1).

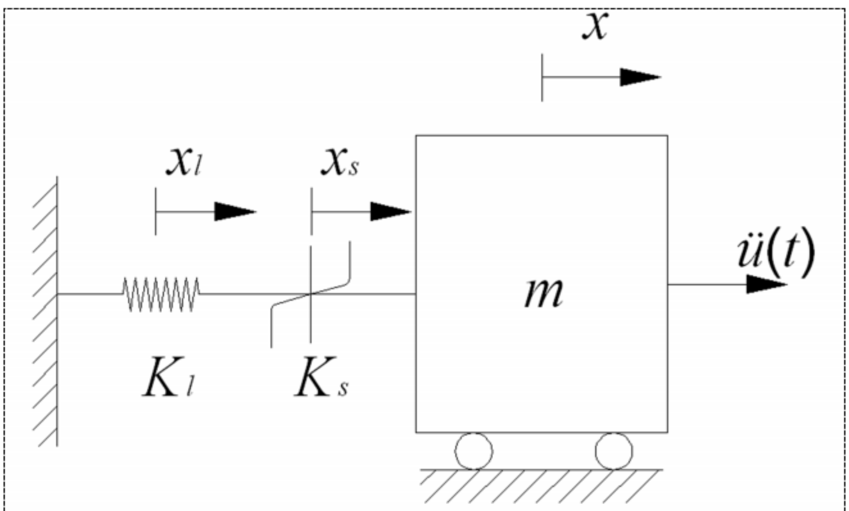

Figure 1. System with slip spring.

The reported slip approximation function (Figure 2) is obtained inverting a sigmoid function, which is appropriate to describe the slip or pinching behaviour. In the classical formulation of the sigmoid function two parameters are introduced, the slip s and an angular coefficient which describes the inverse of the tangent of the function near origin, defined as k, which may be seen as a tangent stiffness.

It is noteworthy that the slip parameter s does not depend on the dissipated energy of the system as in the original Baber-Noori model. This may be useful, especially in an identification procedure: a structure can show a slip even if dissipation has not occurred yet, or if it has undergone previous unknown degrading events.

The following slip formulation is thus assumed to express displacement as a function of the force in the spring, f:

$$x_s(f) = s\frac{\left(1-e^{-k\cdot f}\right)}{\left(1+e^{-k\cdot f}\right)} \tag{1}$$

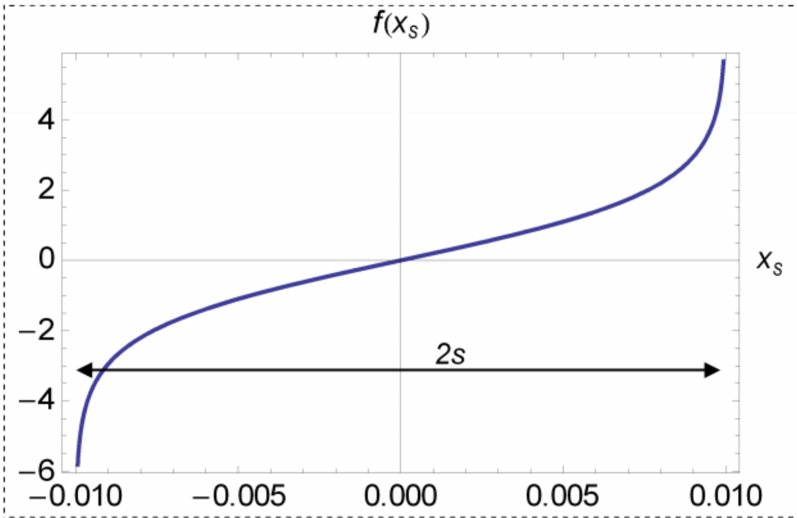

Figure 2. Slip function

Accordingly, the global formulation of the model is been expressed in terms of flexibility, presuming that the global displacement is the sum of an underlying linear component and a non-linear component, i.e. the slip effect:

$$x = x_l + x_s \tag{2}$$

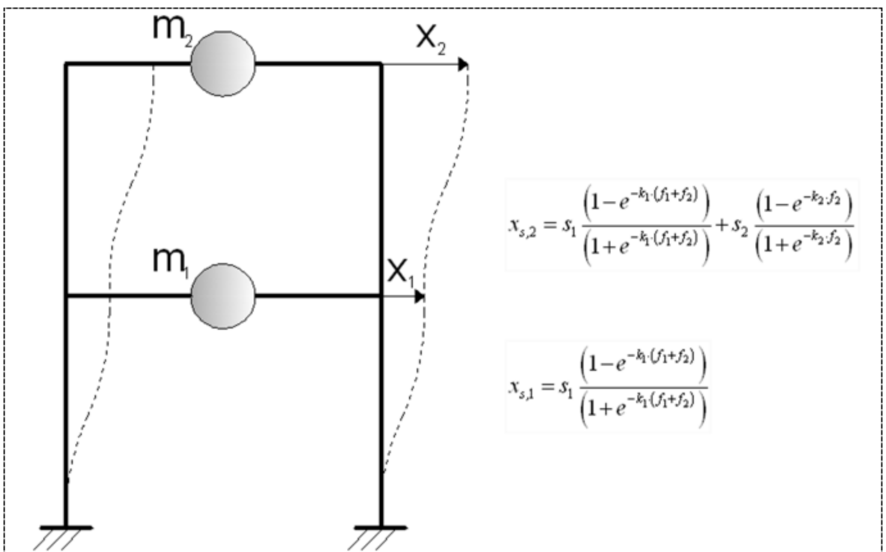

Figure 3. 2 storeys frame with expressions of slip components at the two floors.

Considering a multi-storeys frame (Figure 3), it is possible to express the global displacements in terms of linear and non-linear components. For a 2 storeys frame, one may write the following formulation in terms of flexibility:

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = [\mathbf{Fl}]_{\mathbf{el}} \cdot \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} + \begin{Bmatrix} x_{s,1}(f_1+f_2) \\ x_{s,2}(f_2) + x_{s,1}(f_1+f_2) \end{Bmatrix} \tag{3}$$

where $[\mathbf{Fl}]_{\mathbf{el}}$ is the flexibility matrix of the underlying linear system.

Deriving respect to time equation (3) one obtains:

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = [\mathbf{Fl}]_{\mathbf{el}} \cdot \begin{Bmatrix} \dot{f}_1 \\ \dot{f}_2 \end{Bmatrix} + \begin{bmatrix} \dfrac{2e^{k_1(f_1+f_2)}k_1 s_1}{\left(1+e^{k_1(f_1+f_2)}\right)^2} & \dfrac{2e^{k_1(f_1+f_2)}k_1 s_1}{\left(1+e^{k_1(f_1+f_2)}\right)^2} \\ \dfrac{2e^{k_1(f_1+f_2)}k_1 s_1}{\left(1+e^{k_1(f_1+f_2)}\right)^2} & \dfrac{2e^{k_2 f_2}k_2 s_2}{\left(1+e^{k_2 f_2}\right)^2} + \dfrac{2e^{k_1(f_1+f_2)}k_1 s_1}{\left(1+e^{k_1(f_1+f_2)}\right)^2} \end{bmatrix} \begin{Bmatrix} \dot{f}_1 \\ \dot{f}_2 \end{Bmatrix} \tag{4}$$

The second part of equation (4) clearly assumes the form of a non-linear flexibility matrix $[\mathbf{Fl}]_{\mathbf{s}}$ due to the slip behaviour. The two flexibility matrixes can be summed and then inverted in order to evaluate the tangent stiffness matrix $[\tilde{\mathbf{K}}(\mathbf{f})]$ of the coupled linear and slip springs. Therefore the equations of motion associated to the systems assume the following form:

$$\begin{cases} [\mathbf{M}] \cdot \{\ddot{\mathbf{x}}\} + \{\mathbf{f}\} = -[\mathbf{M}]\{\ddot{\mathbf{u}}_g\} \\ \{\dot{\mathbf{f}}\} = \left[\tilde{\mathbf{K}}(\mathbf{f})\right] \cdot \{\dot{\mathbf{u}}\} \\ \left[\tilde{\mathbf{K}}(\mathbf{f})\right] = \left[[\mathbf{Fl}]_{\mathbf{el}} + [\mathbf{Fl}]_{\mathbf{s}}\right]^{-1} \end{cases} \tag{5}$$

where $[\mathbf{Fl}]_{\mathbf{s}}$ is the Jacobian matrix related to the slip function, $[\mathbf{Fl}]_{\mathbf{el}}$ is the flexibility matrix of the underlying linear system and the matrix $[\tilde{\mathbf{K}}(\mathbf{f})]$ is the matrix of tangent stiffness of the nonlinear system.

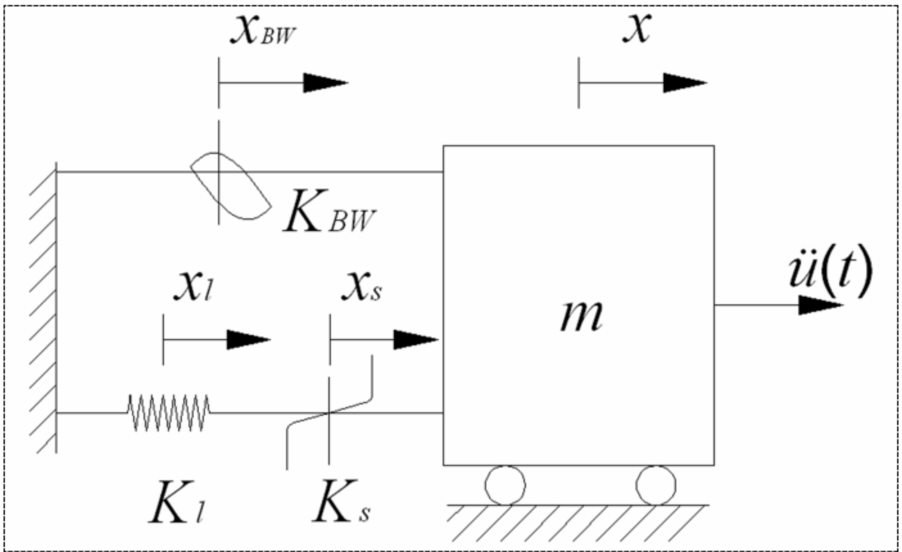

Figure 4. Model of hysteretic system with slip.

In order to consider also hysteresis, a Bouc-Wen model is then introduced, whose purely hysteretic part is assumed to be in parallel with the slip and the linear spring (Figure 4). The new formulation is written as follows:

$$\begin{cases} [\mathbf{M}]\cdot\{\ddot{\mathbf{x}}\}+\{\mathbf{f}\}=-[\mathbf{M}]\{\ddot{\mathbf{u}}_g\} \\ \{\dot{\mathbf{f}}\}=\left[\tilde{\mathbf{K}}(\mathbf{f})\right]\cdot\{\dot{\mathbf{u}}\}+\{\dot{\mathbf{f}}\}_{hyst} \\ \left[\tilde{\mathbf{K}}(\mathbf{f})\right]=\left[[\mathbf{Fl}]_{\mathbf{el}}+[\mathbf{Fl}]_s\right]^{-1} \end{cases} \tag{6}$$

where the hysteretic part of the restoring force has the following explicit form:

$$\{\dot{\mathbf{f}}\}_{hyst}=\begin{Bmatrix} -|f_1+f_2|^{n_1}\cdot\left[\beta_1 sign\left((f_1+f_2)\dot{x}_1\right)+\gamma_1\right]\dot{x}_1+|f_2|^{n_2}\cdot\left[\beta_2 sign\left(f_2(\dot{x}_2-\dot{x}_1)\right)+\gamma_2\right](\dot{x}_2-\dot{x}_1) \\ -|f_2|^{n_2}\cdot\left[\beta_2 sign\left(f_2(\dot{x}_2-\dot{x}_1)\right)+\gamma_2\right](\dot{x}_2-\dot{x}_1) \end{Bmatrix} \tag{7}$$

In equation (7) the parameters β, γ and n have the usual meaning of the Bouc-Wen model, and the subscripts denote the DoF.

3. Dynamic identification of a 2DoF frame with hysteresis and slip

Let us consider a 2-storey building frame and assume that its motion occurs essentially in one direction. Table 1 reports the lumped mass matrix as well as the stiffness terms. Hence, the modal frequencies associated to the linear part of the system are 4.00 Hz and 11.46 Hz, respectively.

Table 1. Mass and stiffness terms.

m_1 [kg]	m_2 [kg]	K_{11} [N/m]	K_{12} [N/m]	K_{22} [N/m]
16300	16000	6.69e7	-3.13e7	2.74e7

Based on the set of equations (6), non-linearity was introduced in the system. Hysteresis and slip parameters were identified from two types of dynamic response signals: the acceleration ideally induced by a sweep-sine excitation applied at the support and that ideally produced by the El Centro earthquake accelerogram, respectively [10].

3.1. Sweep-sine excitation

The first numerical example refers to a system excited at the foundation by a sweep-sine loading that goes from 3 rad/s to 5 rad/s in 10 s. The response of the system is obtained by integrating equation (6) via an explicit Runge-Kutta method. The parameters that characterize the non-linearity of the system are given in table 2, which reports the exact values for the parameters supposed to be unknown. The parameters k_1 and k_2 characterise the type of slip law and, in practical applications, may be set based on experimental evidence. In this example they were set both to 1e-5 N/m.

Table 2. Exact values of hysteresis and slip terms.

β_1	β_2	γ_1	γ_2	n_1	n_2	s_1 [m]	s_2 [m]
5	3	2	2	1	1	2e-3	2e-3

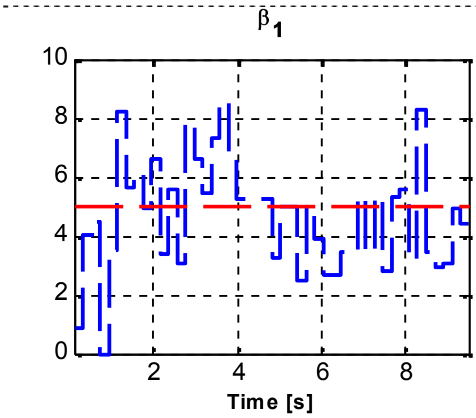

Figure 5. Identified β_1 parameter, in red the exact value, in blue the identified one.

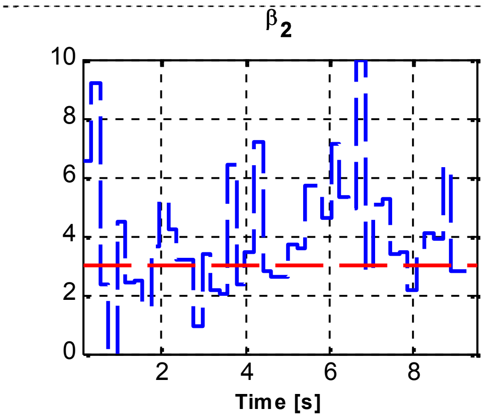

Figure 6. Identified β_2 parameter, in red the exact value, in blue the identified one.

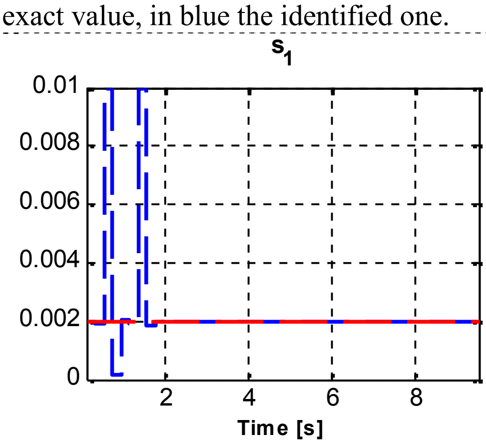

Figure 7. Identified s_1 parameter, in red the exact value, in blue the identified one.

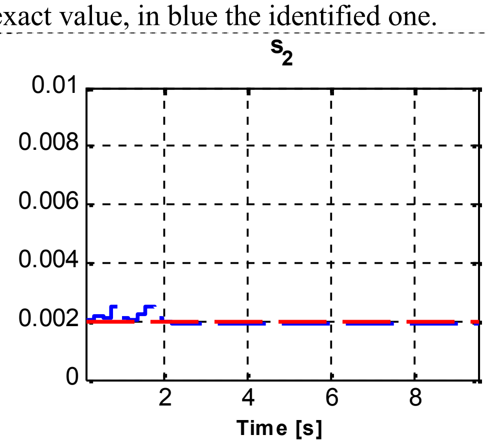

Figure 8. Identified s_2 parameter, in red the exact value, in blue the identified one.

The set of figures 5-8 show the main non-linear terms that have been estimated, namely β and s terms. The slip terms s are identified almost to their exact values and the identification results show their consistency in time. Punctual estimates for Bouc-Wen term β, instead, appear to be less accurate, but they apparently fluctuate around the true value of the parameter, for both degrees of freedom.

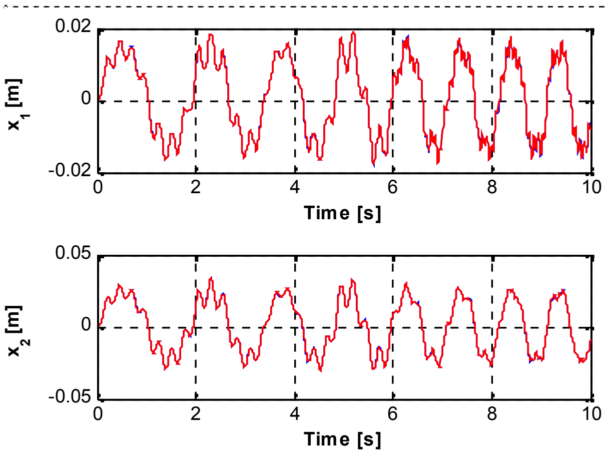

Figure 9. Simulated response in terms of displacements (in blue), identified response (in red).

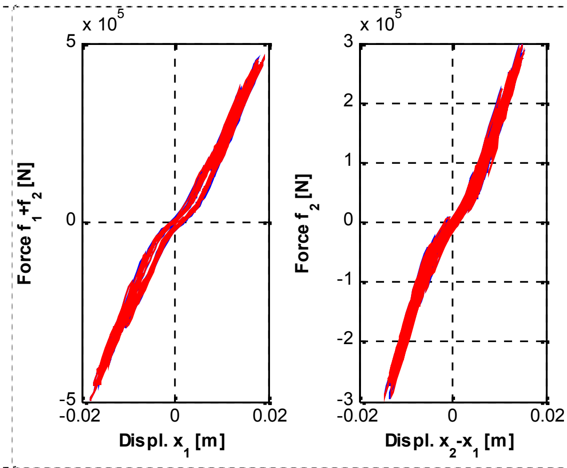

Figure 10. Phase plot of displacements vs. hysteretic force. Simulated in blue, identified in red.

Figure 9-10 show how the response produced by identified parameters matches almost perfectly the original response. Figure 10 emphasize the non-linear behaviour of the system; in fact one can notice the slip effect near the origin, though the dissipation is relatively small.

3.2. Frame subject to earthquake excitation

The second example proposed refers the same underlying linear system, with different values of non-linear parameters to be estimated, as reported in table 3. In this case, the system is excited by the El Centro accelerogram (sampling frequency: 100 Hz).

Table 3. Exact values of hysteresis and slip terms.

β_1	β_2	γ_1	γ_2	n_1	n_2	s_1 [m]	s_2 [m]
30	10	2	2	1	1	3e-3	2e-3

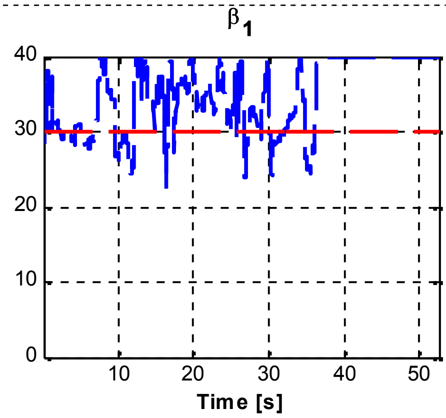

Figure 11. Identified β_1 parameter, in red the exact value, in blue the identified one.

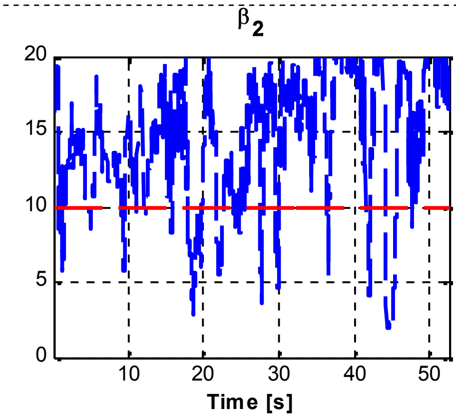

Figure 12. Identified β_2 parameter, in red the exact value, in blue the identified one.

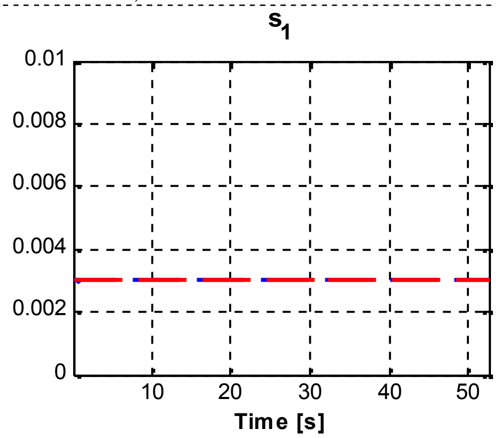

Figure 13. Identified s_1 parameter, in red the exact value, in blue the identified one.

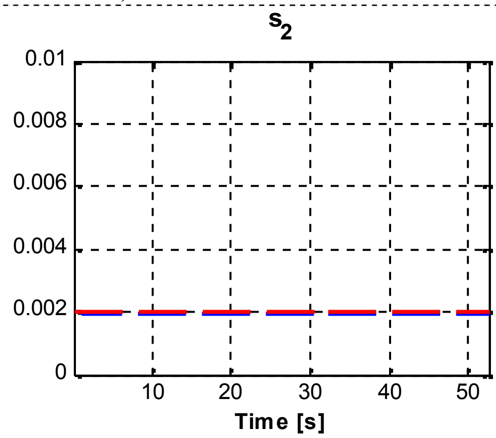

Figure 14. Identified s_2 parameter, in red the exact value, in blue the identified one.

The estimation of the main non-linear parameters is very accurate for the slip terms, whilst the β coefficients oscillate considerably in time.

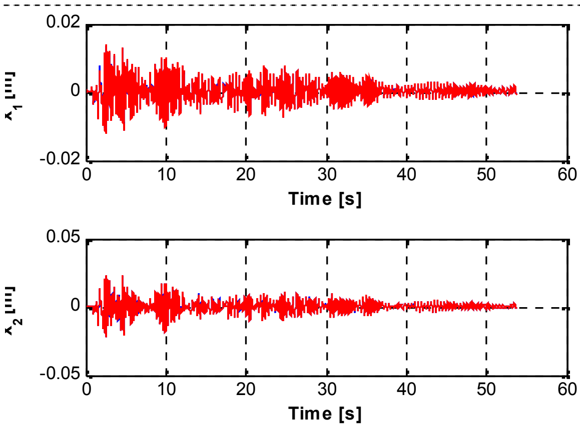

Figure 15. Simulated response in terms of displacements (in blue), identified response (in red).

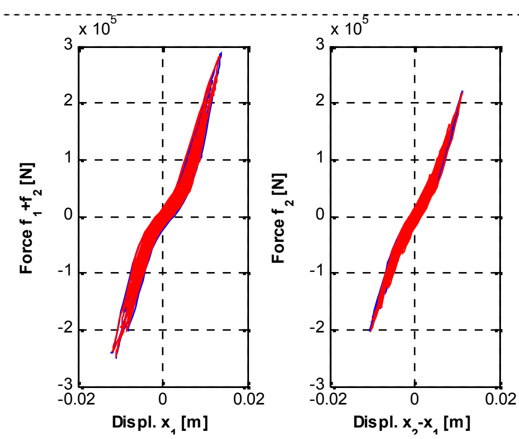

Figure 16. Phase plot of displacements versus hysteretic force. Simulated in blue, Identified in red.

Figure 15-16 show the fitting of the response of the system using the punctual values of the coefficients. Also in this example, the identified response matches almost perfectly the response of the system. Figure 16 highlights the non-linear behaviour of the system.

The mean values of the estimates, which are reported in table 4, demonstrated to be sufficient to obtain a good predictive capacity, through numerical integration. In fact, Figure 17 shows a comparison between the exact response and that predicted by the system with the identified parameters listed in table 4.

Table 4. Identified non-linear terms (mean value).

β_1	β_2	γ_1	γ_2	n_1	n_2	s_1 [m]	s_2 [m]
34.82	14.52	1.26	2.55	1.00	1.00	3.03e-3	1.99e-3

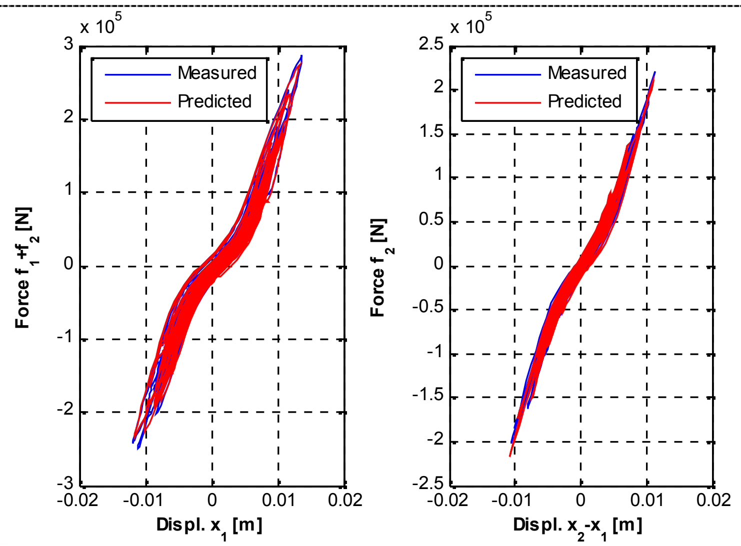

Figure 17. Comparison of the phase plot between data and prediction.

4. Conclusions

This paper has presented a parametric approach to the non-linear identification of framed hysteretic systems with slip. To this end, a modified Bouc-Wen Baber-Noori model was proposed for simulating pinching.

The structure's global dynamic properties were identified numerically in two different loading conditions: a sweep-sine excitation applied at the support and the El Centro earthquake. As a result, the identified system was able to reproduce satisfactorily the non-linear and hysteretic behaviour of the frame.

Future developments of the method shall focus on the identification of degrading systems under seismic excitation. The final aim is the application to experimental data and the realisation of an archive including a wide set of non-linear models.

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